Tytuł projektu

Kombinatoryka Littlewooda-Richardsona i analiza spektralna Coxetera w badaniu algebr Liego oraz problemów typu Birkhoffa

Project title

Littlewood-Richardson combinatorics and Coxeter spectral analysis in the study of Lie algebras and Birkhoff type problems

Dyscyplina /Area of science

matematyka

PROJECT DESCRIPTION

Project goals

- To apply Littlewood-Richardson combinatorics for investigation of invariant subspaces of nilpotent linear operators
- To understand relationships between quadratic forms and infinite dimensional Lie algebras
- To develop Coxeter spectral study of signed graphs, edge-bipartite graphs, their Coxeter groups and related mesh generalized root systems
- To apply obtained results to the classification of derived categories of a class of algebraic singularities and weighted projective lines, up to the usual triangle equivalences.

Outline

The main aim of the doctoral research project is to solve several open problems of representation theory, spectral graph theory and categorical algebra inspired by the classical study of Klein singularities and Birkhoff type problems for nilpotent linear operators in close relation with

- corresponding Littlewood-Richardson combinatorics,
- quadratic forms and their connection with a class of infinite-dimensional Lie algebras,
- Coxeter spectral study of signed graphs, edge-bipartite graphs, their Coxeter groups and related mesh generalized root systems,
- applications of obtained results to the classification of derived categories of a class of algebraic singularities and weighted projective lines, up to the usual triangle equivalences

This is a well defined cooperative research project supervised by two closely cooperating researchers: J. Kosakowska and D. Simson, studying a large class of Birkhoff type problems for nilpotent linear operators by means of different combinatorial tools having a common root; namely, the combinatorics of the irreducible root systems in the sense of Bourbaki and their Coxeter-Weyl groups, with applications in Lie theory and singularity theory.

On one hand, the research direction of D. Simson is partially a continuation of the research project NCN DEC-2012/03/B/ST1/00824 pt. Oswojoność problemów macierzowych i koalgebr, problemy Birkhoffa i sieciowe systemy pierwiastków, 2012-2016 (see also the publications [S1] and [S2] listed below), extended by recent applications of the mesh root system technique (introduced by Simson over 10 years ago) to the Coxeter spectral classification of positive signed graphs. One of our aims is to extend the algorithmic classification given in [S3] and [S4] for edge-bipartite graphs with loops, to positive connected signed graphs without loops. In particular, given a Coxeter polynomial F(t)$\in Z[t]$ of such a signed graph Δ one should construct a unique finite Cox_Δ-mesh geometry of roots of Δ that defines an algorithmic formula computing the strong Gram congruence with the canonically chosen bigraph D_F such that its Coxeter polynomial coincides with F(t). On this way we get an algorithmic computational tool for deciding if the bilinear Euler characteristic $\chi_R: Z^n \times Z^n \to Z^n$ of a finite-dimensiona algebra R determines the bounded derived category $D^b(mod\ R)$ of the R-module category $mod\ R$, up to the triangle equivalences of derived categories.

On the other hand, the research direction of J. Kosakowska is partially a continuation of her cooperation with Markus Schmidmeier from Florida Atlantic University and her PhD student Mariusz Kaniecki (see the publications [K1]-[K4] listed below). One of tasks of this doctoral research project is to develop Littlewood-Richardson tableaux and other combinatorial tools in the theory of invariant subspaces of nilpotent linear operators. In particular we want to investigate properties of new combinatorial tools (i.e. socle tableaux that were defined recently by Kosakowska and Schmidmeier), compare them with Littlewood-Richardson tableaux and apply them to describe algebraic and geometric properties of invariant subspaces (e.g. to generalize results of [K3]). We also want to associate a Lie algebra (using standard Ringel-Hall algebra theory) with a class of invariant subspaces and compare it with a Lie algebra associated with suitably chosen quadratic form.

Work plan

1. Investigate various types of tableaux and their connections with invariant subspaces of nilpotent linear operators
2. Associate with a given bilinear form an infinite dimensional Lie algebra and investigate its properties
3. Develop Coxeter spectral study of signed graphs, edge-bipartite graphs, their Coxeter groups and related mesh generalized root systems
4. Try to apply obtained results to the classification of derived categories of a class of algebraic singularities and weighted projective lines, up to the usual triangle

equivalences

Literature

[K1] Justyna Kosakowska and Markus Schmidmeier, The boundary of the irreducible components for invariant subspace varieties, Mathematische Zeitschrift, 290 (2018), 953-972.

[K2] Mariusz Kaniecki, Justyna Kosakowska and Markus Schmidmeier, Operations on Arc Diagrams and Degenerations for Invariant Subspaces of Linear Operators. Part II , Communications in Algebra 46 (2018), 2243-2263.

[K3] Justyna Kosakowska and Markus Schmidmeier, Box moves on Littlewood-Richardson tableaux and an application to invariant subspace varieties, Journal of Algebra 491 (2017), 241-264.

[K4] Justyna Kosakowska and Markus Schmidmeier, Operations on arc diagrams and degenerations for invariant subspaces of linear operators, Tran. Amer. Math. Soc. 367, (2015), 5475-5505.

[S1] Daniel Simson, Flat complexes, pure periodicity and pure acyclic complexes, Journal of Algebra, 480 (2017), 298-308.

[S2] Daniel Simson, Representation-finite Birkhoff type problems for nilpotent linear operators, J. Pure Appl. Algebra 222 (2018), 2181-2198.

[S3] Daniel Simson, A Coxeter spectral classification of positive edge-bipartite graphs I. Dynkin types B_n, C_n, F_4, E_6, E_7 and E_8 , Linear Algebra Appl. 557(2018), 105-133.

[S4] Daniel Simson, Symbolic computations of strong Gram congruences for Cox-regular positive edge-bipartite graphs with loops, Linear Algebra Appl. 573 (2019), 90-143.

Required initial knowledge and skills of the PhD candidate

- ➔ Analytical thinking
- ➔ Understanding of basic mathematics
- ➔ Understanding of algebra (with emphasis on linear algebra)
- ➔ Knowledge about abstract root's systems and Coxeter spectral study of signed graphs
- ➔ Eager to work hard

Zgłaszający projekt/ Author of the project

dr hab. Justyna Kosakowska prof. dr hab. Daniel Simson stopień/tytuł, imię, nazwisko	justus@mat.umk.pl simson@mat.umk.pl e-mail

	Zakład Kombinatoryki i Obliczeń Symbolicznych, Wydział Matematyki i Informatyki, UMK jednostka organizacyjna
Proponowani promotorzy i mentorzy/prospective supervisors	
1) promotor główny/ main supervisior	
dr hab. Justyna Kosakowska prof. dr hab. Daniel Simson	e-mail justus@mat.umk.pl simson@mat.umk.pl
	Zakład Kombinatoryki i Obliczeń Symbolicznych, Wydział Matematyki i Informatyki, UMK jednostka organizacyjna
2) promotor pomocniczy / co-supervisor	