Tytuł projektu

Dynamika nieliniowych równań różniczkowych cząstkowych na R^N – podejście topologiczne

Project title

Dynamics of Nonlinear Partial Differential Equations on R^N – Topological Approach

Dyscyplina /Area of science

Nauki matematyczne

PROJECT DESCRIPTION

Project goals

- To find criteria under which the Schrödinger equation with Kato-Rellich type potential admits stationary solutions at resonance, especially when the nonlinear term is a bounded function.
- To provide criteria for asymptotic bifurcation of solutions of the Schrödinger equation with Kato-Rellich type potentials.
- To detect connecting orbits between stationary solutions of hyperbolic equations on R^N.
- To provide criteria for the existence of periodic solutions of hyperbolic equations on R^N with damping in both resonant and non-resonant cases when the nonlinear perturbation is time dependent.

Outline

The subject of study are evolution partial differential equations on the whole Euclidean space R^N. Special interest will be put on problems arising in search of standing wave solutions and its profiles of the externally driven nonlinear Schrödinger equation as well as connecting orbits and periodic solutions of parabolic and hyperbolic problems on R^N involving a potential V and nonlinear term f. Properties of such problems and their topological structure hinge strongly on the spectrum of the operator $-\Delta+V$, which in turn is determined by the type of potential V. As for the nonlinearity f, one assumes that either some geometric or asymptotic conditions involving the spectrum of $-\Delta+V$ hold, for example: the Landesman-Laser type conditions, sign conditions or asymptotic linearity with the limit value missing the spectrum of $-\Delta+V$.

Some effective methods of studying such problems are based on Conley type homotopy/homology indices or topological degrees, which are algebraic topology invariants of semiflows/equations possessing some compactness properties. Compactness restricts the "space", in which nonlinear systems can be deformed preserving their homotopy invariants. In general, Conley type index is applicable to equations which are neither dissipative nor possessing an attractor (see [10]). Here the parabolic semiflow (when studying profiles of standing waves or stationary solutions) or the related hyperbolic one will be considered. An appropriate topological degree would be the Euler characteristics of the Conley type index for a properly chosen isolating block.

In both parabolic and hyperbolic cases, the key issues, before applying either a Conley type index or topological degree, will be compactness properties, i.e. that any bounded subset of the phase space is admissible with respect the semiflow or that one can approximate the stationary solution problem by use of a family of finite dimensional ones. In special cases, when *V* is of Kato-Rellich type, one may use the so-called tail estimate techniques in order to gather compactness properties sufficient for applying homotopy invariant. It was done so in some cases for parabolic semiflows under the assumption that *f* is bounded by a square integrable function (see e.g. [8], [1], [2], [3], [4] and [5]). However, in a quite common case when *f* is just bounded (see e.g. [6], [8] ad [11]) it is not clear how to show the admissibility of the considered semiflows even if *V* is of Kato-Rellich type. Some special estimates on the norm of solutions are necessary here. They should provide methods for studying existence, connecting orbits or bifurcation problems related to the parabolic semiflow. An open question about the admissibility and following implications appears also when the kernel of the linearization of the right hand side is finite dimensional but zero lies in a spectral gap, i.e. between two essential parts of the spectrum $-\Delta+V$. It is natural when *V* is bounded and changes sign. Then the subspace associated to the essential part of the spectrum of the linearized right hand side intersected with the negative halfline is infinite dimensional. It causes topological problems, since the norm topology in that part of the spectrum is too big to keep the homotopy invariants constant along deformations. Either a homotopy index with modified topology or a homotopy index introduced by Izydorek and Rybakowski in [9] could be effective in such situations.

Work plan

1. Finding estimates for the parabolic semiflow generated by the nonlinearly perturbed Schrödinger operator and identifying the admissibility type of the semiflow.
2. Finding estimates for the hyperbolic semiflow generated by the nonlinearly perturbed Schrödinger operator and identifying the admissibility type of the semiflow.
3. Studying the structure of all full bounded solutions to the parabolic and hyperbolic equations in both resonant and nonresonant cases.
4. Computing Conley type indices for the parabolic and hyperbolic semiflows at infinity for nonresonant case and proving the existence of nontrivial full bounded solutions in the resonant case.

5. Computing the fixed point index of the Poincaré translation along trajectories operator for the hyperbolic equation.

Literature

[1] A. Ćwiszewski, R. Łukasiak, *A Landesman-Lazer type result for periodic parabolic problems on R^N at resonance*, Nonlinear Anal. 125 (2015), 608_625.
[2] A. Ćwiszewski, R. Łukasiak, *Forced periodic solutions for nonresonant parabolic equations on R^N,* submitted for publication.
[3] A. Ćwiszewski, W. Kryszewski, *Bifurcation from infinity for elliptic problems on R^N*, Calc. Var. Partial Differential Equations 58 (2019), no. 1, Art. 13, 23 pp.
[4] A. Ćwiszewski, P. Kokocki, *Stationary solutions and connecting orbits for resonant parabolic equations on R^N*, preprint (2019).
[5] A. Ćwiszewski, P. Kokocki, *Stationary solutions and connecting orbits for resonant hyperbolic equations on R^N*, preprint (2019).
[6] F. Genoud, *Global bifurcation for asymptotically linear Schrödinger equations*, Nonlinear Differ. Equ. Appl. 20, 23_35 (2013).
[7] M. Izydorek, K.P. Rybakowski, *Conley index in Hilbert spaces and a problem of Angenent and Van der Vorst*, Fund. Math. 173 (2002), no. 1, 77-100.
[8] W. Kryszewski, A. Szulkin, *Bifurcation from infinity for an asymptotically linear Schrödinger equation*, J. Fixed Point Theory Appl. 16 (12), 411-435 (2014).
[9] M. Prizzi, *On admissibility of parabolic equations in R^N*, Fund. Math. 176, 261-275 (2003).
[10] K.P. Rybakowski, *The homotopy index and partial differential equations*, Universitext, Springer, Berlin (1987).
[11] C. A. Stuart, *Asymptotic bifurcation and second order elliptic equations on R^N*, Ann. Inst. H. Poincaré Anal. Non Linéaire 32(6), 1259-1281 (2015).

Required initial knowledge and skills of the PhD candidate

- ➔ Analytical thinking
- ➔ Willingness to self-study
- ➔ Understanding of mathematical analysis
- ➔ Basic knowledge of general topology, functional analysis and partial differential equations

Zgłaszający projekt/ Author of the project

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Proponowani promotorzy i mentorzy/prospective supervisors

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