

<b>Tytuł projektu</b>
Uogólnienia kryterium realignmentu i powiązanych kryteriów splątania do przypadku wielocząstkowego
<b>Project title</b>
<b>Generalizing realignment criterion and related entanglement criteria to multipartite scenario</b>
<b>Dyscyplina /Area of science</b>
Nauki fizyczne
<b>PROJECT DESCRIPTION</b>
<p><b>Project goals</b></p> <ul style="list-style-type: none"> <li>• To find new multidimensional analogs of matrix norms and invariants</li> <li>• To compare different generalisations of singular value decomposition as tools for detecting multipartite entanglement</li> <li>• To look for physical interpretations of the above norms and multidimensional matrix invariants from the point of view of quantum information protocols.</li> </ul> <p><b>Outline</b></p> <p>In Quantum mechanics states are represented by positive, trace-class operators, having trace normalized to identity, acting on a complex Hilberts space, known as <i>density operators</i> (or density matrix, when the basis is fixed). Hilbert space of a composite system is a tensor product of the Hilbert spaces of the subsystems and a product state - when there is no correlations between subsystems - is given by a tensor product of the density operators of subsystems:</p> $\rho = \rho^{(A)} \otimes \rho^{(B)}$ <p>If it is possible to prepare such states locally (due to the lack of correlations). We can produce a correlated state mixing the product states (for example one can add in the state preparation protocol tossing a dice to choose a product state to prepare):</p> $\rho = \sum_i p_i \rho_i^{(A)} \otimes \rho_i^{(B)}$ <p>States of the above form are called <i>separable states</i> and form a proper subset of the set of states - not all quantum states using such protocols. Quantum mechanics, being a non-kolmogorov probability theory, admits correlations between subsystems of a bigger system, which cannot be explained by classical theory. We call states with such correlations <i>entangled states</i>. Entanglement first have been treated as a proof of non-completeness of quantum mechanics, but after they have been shown experimentally, physicists accepted the quantum mechanics as it is, despite its counter-intuitivity. Now there exist quantum-information protocols, like quantum cryptography and quantum teleportation which uses non-classical properties of quantum correlations. Quantum entanglement become a resource for these protocols. Problems of entanglement</p>

detection, its dynamics prediction or how to preserve it become of growing importance from the point of view of possible applications.

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First of the mentioned problems - detecting entanglement between two (in the simplest case) subsystems of a given system is hard even if the state of the system is explicitly given as a density matrix. Because entanglement is the key resource for the novel information protocols, various necessary and sufficient conditions for the state to be entangled has been developed. It is worth to remind here, that in the real-life scenarios a density matrix is not given - one can reconstruct it in the process of \textit{quantum tomography}, which requires a lot of state-destructing measurements to perform (the number of measurements grows like with the dimension of the Hilbert space and like with the desired accuracy). Hence methods of detecting entanglement without full tomography and optimising the number of measurements are of great interest.

$(n \cdot m)^2$

The theory of convex sets / convex cones shows, that for any entangled state there exists an observable (measurable quantity, analogon of measurable function in the classical probability theory) positive on all separable states but negative in the given entangled state. We call such an observable an *entanglement witness*. To detect entanglement of a system one has to measure one global observable of the system, not of them which would be necessary for the full system tomography. Measuring one appropriately chosen entanglement witness we can detect entanglement of a given state without knowing its density matrix.

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Except for the lowest dimensions of the subsystems, the classification of entanglement witnesses is not known (otherwise the problem of entanglement would be solved). The famous Jamiołkowski isomorphism relate entanglement witnesses between positive, but not completely positive maps between matrix algebras (or algebras in general) and in this way separability problem is related to another mathematical problem - classification of positive maps.

Another important entanglement criterion is so called *realignment criterion*, *cross-norm criterion* or *CCNR criterion* (and its various generalisations).

The project will concentrate on generalising realignment criterion and related entanglement criteria to multipartite scenario. The trace norm plays central role in the criterion and its features connect the criterion with a family of positive maps and entanglement witnesses. The definition of the trace norm uses singular value decomposition of a matrix, known also in the context of quantum mechanics as the Schmidt decomposition. A number of publication announce non-existence of Schmidt decomposition in the multipartite case. In is true, when one expects the Schmidt form of a pure state to be like a GHZ state. There are although papers (at least for qubits) calculating dimensionality of the space of local orbits in the set of pure states and finding invariants of the local unitaries group action (analogues of Schmidt coefficients). On the other hand, a various generalisations of the Singular Value Decomposition, known as Higher Order Singular Value Demposition, have been developed in the Data Science as a

form of Principal Component Analysis for multidimensional data.

The goal of the PhD studies would be to look for a possible generalisation of the trace norm for multidimensional matrices and then to find a multipartite generalisation of the realignment criterion and the related criteria and looking for the corresponding entanglement witnesses in the multipartite scenario.

Under my supervision, a candidate will be expected to get familiar during the PhD studies with:

- Theory of positive maps between matrix algebras, especially for detecting quantum entanglement.
- Theory of entanglement witnesses, Bell inequalities and detecting correlations in the distant-laboratories scenario.
- Theory of multipartite entanglement.
- Theory of Lie groups their actions, sets of orbits and their polynomial invariants.
- Matrix norms, theory of multidimensional matrices and their invariants
- Algorithms of Data Science for multidimensional data sets.
- Methods of numerical verification of new hypotheses.

#### Literature

- R.Bhatia *Positive Definite Matrices* (Princeton University Press, 2015)
- M.A.Nielsen, I.L.Chuang *Quantum Computation and Quantum Information* (Cambridge University Press, 2011)

#### Required initial knowledge and skills of the PhD candidate

- ➔ Basic knowledge of algebraic and analytical methods in quantum physics (operator algebras, functional analysis),
- ➔ Eager to learn
- ➔ Basic knowledge of quantum information theory
- ➔ Eager to work hard

#### Zgłaszający projekt/ Author of the project

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#### Proponowani promotorzy i mentorzy/prospective supervisors

1) promotor główny/ main supervisor

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2) promotor pomocniczy / co-supervisor	