Group theory and the Pentaquark

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Abstract. The group classification of exotic pentaquarks involving four standard quarks and a single antiquark to produce strangeness $\mathcal{S} = +1$ is outlined.

21 July 2003

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1. Introduction

. The possibility of observable exotic multiquark states has been the subject of speculation for over three decades and has generated a large literature, a miniscule sample might include[1, 2, 3, 4, 5]. The MIT bag model played a central role in many calculations[1]. Searches for exotic multiquark resonances have been largely unsuccessful in spite of efforts over three decades. Among the possible resonances have been those associated with the pentaquarks involving four quarks and a single antiquark, their existence having been speculated more than three decades ago[3, 6, 7]. Evidence for a pentaquark with strangeness $\mathcal{S}= +1$ has been recently reported by three independent experimental groups[8, 9, 10] involving three different types of experiments..

Herein we discuss the group classification of the states of pentaquarks. Throughout we label irreducible representations of the relevant SU_n groups by appropriate partitions[11, 12]. The usual u, d, s quarks have spin $S = \frac{1}{2}$ with colour corresponding to the $\{1\}^c$ irreducible representation of the group SU_3^c and flavour corresponding to the $\{1\}^f$ irreducible representation of SU_3^{fl}. The complete set of the basic quarks transform as the vector irreducible representation $\{1\}$ of the group SU_{18}[5]. A system of N quarks will span the totally antisymmetric irreducible representation $\{1^N\}$ of SU_{18} which may be decomposed under the group chain

$$SU_{18}^q \rightarrow SU_2^S \times SU_3^c \times SU_3^{fl} \tag{1}$$

A system of N' antiquarks $\bar{q}$ will span the antisymmetric conjugate irreducible representation $\{\overline{1^{N'}}\}$ of another group $SU_{18}^{\bar{q}}$ which may be decomposed under the group chain

$$SU_{18}^{\bar{q}} \rightarrow SU_2^S \times SU_3^c \times SU_3^{fl} \tag{2}$$

The totally antisymmetric states formed by configurations of N quarks and N' antiquarks will span the irreducible representation $\{1^{N+N'}\}$ of the group SU_{36} leading to the classification group chain

$$SU_{36} \rightarrow SU_{18}^q \times SU_{18}^{\bar{q}} \tag{3}$$

Hereon we examine the various group decompositions relevant to the pentaquark. We start with the group SU_{36} and systematically reduce the symmetry using the appropriate group decomposition branching rules until finally the quantum numbers labelling the individual pentaquark are determined. It is demonstrated that there is a single pentaquark state of strangeness $\mathcal{S}= +1$ of charge $q = +1$ arising from the combination of a sextet multiplet of quarks and a single strange antiquark. These states form part of a SU_3^{fl} octet, $\{21\}$, and antidecuplet, $\{3^2\}$, with the relevant pentaquark at the apex of the antidecuplet. Other pentaquarks of strangeness $\mathcal{S}= +1$ arise but with uniquely distinguishable quantum numbers.

2. The $SU_{36} \to SU_{18}^{q} \times SU_{18}^{\bar{q}}$ decompositions

Assuming that under $SU_{36} \to SU_{18}^{q} \times SU_{18}^{\bar{q}}$

$$\{1\} \to \{1\}^{q} + \{\bar{1}\}^{\bar{q}} \tag{4}$$

we have for k particles

$$\begin{aligned}\{1^k\} &\to (\{1\}^{q} + \{\bar{1}\}^{\bar{q}}) \otimes \{1^k\} \\ &= \sum_{x=0}^{k} \{1^{k-x}\}^{q} \times \{\bar{1}^{x}\}^{\bar{q}}\end{aligned} \tag{5}$$

where we use $\otimes$ to indicate the operation of plethysm[11, 12, 13, 14]

3. The colour singlet pentaquarks

For 4 quarks and a single antiquark we have just the $SU_{18}^{q} \times SU_{18}^{\bar{q}}$ irreducible representation $\{1^4\}^{q} \times \{\bar{1}\}^{\bar{q}}$ to consider. We first note that for $SU_3^c \times SU_3^{fl}$[15]

$$(\{1\}^{c} \times \{1\}^{fl}) \otimes \{\rho\} = \sum_{\varepsilon \vdash \omega_\rho} \{\varepsilon\}^{c} \times \{\varepsilon \circ \rho\}^{fl} \tag{6}$$

Thus under $SU_{18}^{q} \to SU_2^{S} \times SU_3^{c} \times SU_3^{fl}$ we have

$$\begin{aligned}\{1^4\}^{q} &\to (\{1\}^{S} \times \{1\}^{c} \times \{1\}^{fl}) \otimes \{1^4\} \\ &= \{4\} \times (\{1\}^{c} \times \{1\}^{fl}) \otimes \{1^4\} \\ &\quad + \{31\} \times (\{1\}^{c} \times \{1\}^{fl}) \otimes \{21^2\} \\ &\quad + \{2^2\} \times (\{1\}^{c} \times \{1\}^{fl}) \otimes \{2^2\}\end{aligned} \tag{7}$$

Making repeated use of (6) and replacing the irreducible representations of SU_2^S by the appropriate spin multiplicities $(2S+1)$ as a left superscript, dropping the colour (c) and flavour (fl) superscripts, we have for the complete decomposition

$$\begin{aligned}\{1^4\} \to\ & {}^{5}(\{31\}\{1\} + \{1\}\{31\} + \{2^2\}\{2^2\}) \\ & + {}^{3}(\{4\}\{21^2\} + \{31\}\{31\} + \{31\}\{21^2\} + \{2^2\}\{31\} + \{2^2\}\{1\} \\ & + \{1\}\{4\} + \{1\}\{2^2\} + \{1\}\{21^2\}) \\ & + {}^{1}(\{4\}\{2^2\} + \{31\}\{31\} + \{31\}\{21^2\} + \{2^2\}\{4\} + \{2^2\}\{2^2\} \\ & + \{1\}\{31\} + \{1\}\{21^2\})\end{aligned} \tag{8}$$

The single antiquark transforms as the $\{\bar{1}\}$ irreducible representation of $SU_{18}^{\bar{q}}$ and under (2) decomposes as

$$\{\bar{1}\} \to {}^{2}\{\bar{1}\}^{c} \times \{\bar{1}\}^{fl} \tag{9}$$

To form pentaquark states with 4 quarks and an antiquark we must form the product of the 4 quark states arising from (8) with those of (9). Under the usual postulates of

QCD only colourless states correspond to observables. Thus we seek the pentaquark states that transform under the group SU_3^c as the $\{0\}^c$ irreducible representation. This is only possible for those four quark states that transform as $\{1\}^c$ under SU_3^c. Note that under SU_3^c

$$\{1\}^c \times \{\bar{1}\}^c = \{0\}^c + \{21\}^c \tag{10}$$

Thus from (8) the only possible pentaquark states will come from

$$^5(\{1\}\{31\}) + {}^3(\{1\}\{4\} + \{1\}\{2^2\} + \{1\}\{21^2\}) + {}^1(\{1\}\{31\} + \{1\}\{21^2\}) \tag{11}$$

Recall the SU_3 Kronecker products

$$\{21^2\} \times \{\bar{1}\} = \{0\} + \{21\} \tag{12a}$$
$$\{2^2\} \times \{\bar{1}\} = \{21\} + \{3^2\} \tag{12b}$$
$$\{31\} \times \{\bar{1}\} = \{21\} + \{3\} + \{42\} \tag{12c}$$
$$\{4\} \times \{\bar{1}\} = \{3\} + \{51\} \tag{12d}$$

Combining these with (9) and keeping only colour singlets gives the possible pentaquark states under $SU_2^S \times SU_3^{fl}$ as

$$^{6,4}(\{21\} + \{3\} + \{42\}) \tag{13a}$$
$$^{4,2}(\{0\} + 2\{21\} + \{3\} + \{3^2\} + \{51\}) \tag{13b}$$
$$^{2}(\{0\} + 2\{21\} + \{3\} + \{42\}) \tag{13c}$$

where again the spin multiplicities appear as left superscripts.

4. Quarks and the $SU_3^{fl} \to U_1^Y \times SU_2^I$ decompositions

The standard charge (q), isospin projection (I_z), strangeness ($\mathcal{S}$) and hypercharge (Y) of the quark triplet are tabulated below

quark	q	I_z	$\mathcal{S}$	Y
u	$\frac{2}{3}$	$\frac{1}{2}$	0	$\frac{1}{3}$
d	$-\frac{1}{3}$	$-\frac{1}{2}$	0	$\frac{1}{3}$
s	$-\frac{1}{3}$	0	-1	$-\frac{2}{3}$

Under $SU_3^{fl} \to U_1^Y \times SU_2^I$ the triplet irreducible representation of SU_3^{fl} decomposes as

$$\{1\}^{fl} \to \{\tfrac{1}{3}\} \times \{1\} + \{-\tfrac{2}{3}\} \times \{0\} \tag{14}$$

For an arbitrary irreducible representation $\{\lambda\}^{fl}$ of SU_3^{fl} we have the decompositon

$$\begin{aligned}\{\lambda\}^{fl} &\to (\{\tfrac{1}{3}\} \times \{1\} + \{-\tfrac{2}{3}\} \times \{0\}) \otimes \{\lambda\} \\ &= \sum_{\rho} (\{\tfrac{1}{3}\} \times \{1\}) \otimes \{\lambda/\rho\} \cdot (\{-\tfrac{2}{3}\} \times \{0\}) \otimes \{\rho\}\end{aligned} \tag{15}$$

Only the leading term in (15)

$$(\{\tfrac{1}{3}\}\times\{1\})\otimes\{\lambda\} = \sum_{\varepsilon}\{\tfrac{1}{3}\}\otimes\{\varepsilon\}\cdot\{1\}\otimes\{\varepsilon\circ\lambda\} \\ = \{\frac{\omega_\lambda}{3}\}\times\{\lambda\}, \qquad (16)$$

where the right-hand-side of (16) is an irreducible representation of $U_1^Y \times SU_2^I$, can yield non-strange 4 quark states. Furthermore, for non-strange 4 quark states it is necessary that the the number of parts of the partition (λ) have less than three parts. Thus the irreducible representation $\{21^2\}\equiv\{1\}$ appearing in (8),(11) and (12) cannot yield non-strange 4 quark states. Noting (11) and using (16) we have the relevant non-strange leading terms as below

SU_3^{fl}	$U_1^Y \times SU_2^I$	I	
$\{2^2\}$	$\{\frac{4}{3}\}\times\{0\}$	0	(17a)
$\{31\}$	$\{\frac{4}{3}\}\times\{2\}$	1	(17b)
$\{4\}$	$\{\frac{4}{3}\}\times\{4\}$	2	(17c)

The states associated with the maximal isospin projection in (17a,b,c,d) will be respectively

$$uudd \qquad (18a)$$

$$uuud \qquad (18b)$$

$$uuuu \qquad (18c)$$

Pentaquarks of strangeness $\mathcal{S}= +1$ may be formed by combining each member of each isospin multiplet with a single strange antiquark. The resulting pentaquarks, along with their hypercharge (Y), isospin projection (I_z) and electric charge (q) are given in Table 1.

	I	Y	I_z	q
$\{4\}\bar{s}$	2	2		
$uuuu\bar{s}$			2	+3
$uuud\bar{s}$			1	+2
$uudd\bar{s}$			2	+1
$uddd\bar{s}$			-1	0
$dddd\bar{s}$			-2	-1
$\{31\}\bar{s}$	1	2		
$uuud\bar{s}$			1	+2
$uudd\bar{s}$			0	+1
$uddd\bar{s}$			-1	0
$\{2^2\}\bar{s}$	0	2		
$uudd\bar{s}$			0	+1

Table 1. The strangeness $\mathcal{S}$= +1 pentaquarks.

To continue let us look in more detail at the 18 pentaquark states arising from the product

$$\{2^2\} \times \{\bar{1}\} = \{21\} + \{3^2\} \tag{19}$$

Under $SU_3 \to U_1^Y \times SU_2^I$ we have from (15)

$$\{2^2\} \to \{\tfrac{4}{3}\} \times \{0\} + \{\tfrac{1}{3}\} \times \{1\} + \{-\tfrac{2}{3}\} \times \{2\} \tag{20}$$

This gives a set of 6 states with the following (Y, q, I, I_z) numbers

quarks	Y	q	I	I_z
uudd	$\frac{4}{3}$	$\frac{2}{3}$	0	0
uuds	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2}$
udds	$\frac{1}{3}$	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$
uuss	$-\frac{2}{3}$	$\frac{2}{3}$	1	1
udss	$-\frac{2}{3}$	$-\frac{1}{3}$	1	0
ddss	$-\frac{2}{3}$	$-\frac{4}{3}$	1	-1

The six 4-quark states associated with the $\{2^2\}^{fl}$ sextet are shown in Figure 1.

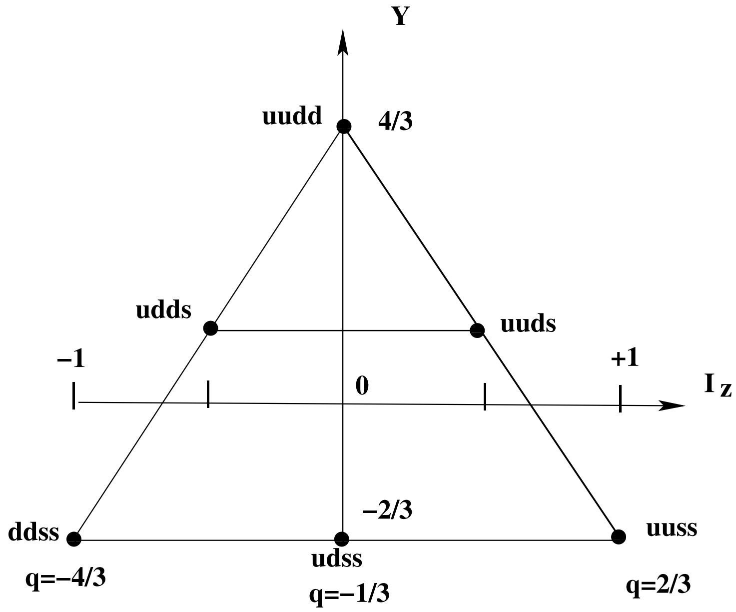

Figure 1. The $\{2^2\}^{fl}$ sextet

These 6 states, each involving 4 quarks, may be combined with the 3 antiquarks ($\bar{u}, \bar{d}, \bar{s}$ to form the 18 pentaquark states associated with the octet and antidecuplet arising in (19). We list below these 18 pentaquarks

penta	I_z	Y	q	$\mathcal{S}$
uudd$\bar{u}$	$-\frac{1}{2}$	1	0	0
uudd$\bar{d}$	$\frac{1}{2}$	1	1	0
uudd$\bar{s}$	0	2	1	$+1$
uuds$\bar{u}$	0	0	0	-1
uuds$\bar{d}$	1	0	1	-1
uuds$\bar{s}$	$\frac{1}{2}$	1	1	0
uuss$\bar{u}$	$\frac{1}{2}$	-1	0	-2
uuss$\bar{d}$	$\frac{3}{2}$	-1	1	-2
uuss$\bar{s}$	1	0	1	-1
udss$\bar{u}$	$-\frac{1}{2}$	-1	-1	-2
udss$\bar{d}$	$\frac{1}{2}$	-1	0	-2
udss$\bar{s}$	0	0	0	-1
udds$\bar{u}$	-1	0	-1	-1
udds$\bar{d}$	0	0	0	-1
udds$\bar{s}$	$-\frac{1}{2}$	1	0	0
ddss$\bar{u}$	$-\frac{3}{2}$	-1	-2	-2
ddss$\bar{d}$	$-\frac{1}{2}$	-1	-1	-2
ddss$\bar{s}$	-1	0	-1	-1

If we make a I_z versus Y plot of the above entries we obtain, as expected, an antidecuplet $\{3^2\}$ on which is superimposed an octet $\{21\}$ with the reported pentaquark state at the apex of the antidecuplet. These states occur with spin $S = \frac{1}{2}$ and $S = \frac{3}{2}$.

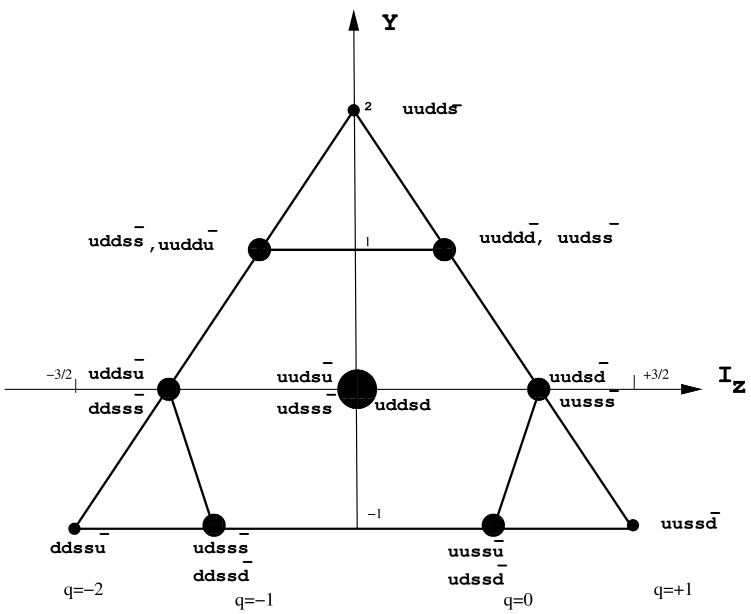

Figure 2. The octet + antidecuplet

5. Concluding remarks

We have shown how the quantum numbers associated with states involving 4 quarks and an antiquark can be readily determined. Nine pentaquarks of strangeness $\mathcal{S}= +1$ and spin $S = \frac{1}{2}$ are found. They may be uniquely distinguished from each other by their isospin (I), isospin projection (I_z) and electric charge (q).

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