However for larger number of qubits, the state that maximized the $n$-tangle is apparently not the N-GHZ state. For instance for $L = 6$, and 8 we get

$$|\phi_6(\pi)\rangle = \frac{1}{4\sqrt{2}}\left(000000 + \hat{\pi}(101000 + 100100 - 110000) + 1 \leftrightarrow 0\right). \tag{35}$$

$$|\phi_8(\pi)\rangle = \frac{1}{4\sqrt{2}}\left(00000000 + \hat{\pi}(00010001 - 01100110 + 10101010 - 00001111 + 01000100) + 1 \leftrightarrow 0\right). \tag{36}$$

There are a total of 32 terms in each state and we have temporarily dispensed with the ket notation.

### B. Transverse field

We now turn on an external field in the transverse direction. This model, the kicked transverse Ising model, has been studied recently as noted above and is also an integrable case [27, 28], and the Jordan - Wigner transformation can be used to diagonalize it. In this case we have

$$|\psi_L(t)\rangle = \left(U_{xx}(J_x)U_{x,z}(B,\pi/2)\right)^t |\psi_L(0)\rangle \tag{37}$$

where $t$ is an integer time, the number of kicks. We now proceed to diagonalize the operator, indicating the key steps. It maybe noted that unlike the treatment in [28] we do not assume the thermodynamic limit, and in this sense the way we solve this problem is also new, though the technique is the same as that for the usual Ising model in a transverse field.

In the kicked transverse Ising spin chain treated here, the Ising interaction is in $x$-direction and the magnetic field is switched on at integer times along the $z$-direction. The first step is to replace the spin variables by Jordan-Wigner fermions through a nonlocal transformation [9]:

$$S_l^+ = \exp\left(i\sum_{n=1}^{l-1} c_n^\dagger c_n\right) c_l^\dagger,\ S_l^z = c_l^\dagger c_l - \frac{1}{2}. \tag{38}$$

The operators $c_l$ and $c_l^\dagger$ obey the usual fermion anticommutation rules. The interaction term in $U_{xx}$ reduces to a combination of nearest-neighbour fermion hopping, pair-fermion annihilation and creation terms on a lattice,

$$U_{xx} = \exp\left(-\frac{iJ_x}{4}\left(\sum_{l=1}^{L-1}(c_l^\dagger - c_l)(c_{l+1}^\dagger + c_{l+1}) - (-1)^{N_F}(c_L^\dagger - c_L)(c_1^\dagger + c_1)\right)\right) \tag{39}$$

where $N_F = \sum_{i=1}^{L} c_i^\dagger c_i$ is the total number of fermions. The last term is due to the periodic boundary condition. The magnetic field term in $U_{x,z}(B,\pi/2)$ becomes a chemical potential term for the total number of fermions. The eigenstates of $U$ will have a definite even or odd fermion number, since $N_f$ commutes with $U$, and we can find the eigenstates in the two sectors separately.

Now, the second step is to Fourier transform through,

$$c_q = \frac{\exp(i\pi/4)}{\sqrt{L}}\sum_{l=1}^{L}\exp(-iql)\, c_l, \tag{40}$$

where the allowed allowed values for $q$ are (taking $L$ to be even)

$$q = \pm\frac{\pi}{L}, \pm\frac{3\pi}{L}, \ldots, \pm\frac{(L-1)\pi}{L} \qquad N_F \text{ even}, \tag{41}$$

$$q = 0, \pm\frac{2\pi}{L}, \pm\frac{4\pi}{L}, \ldots, \pm\frac{(L-2)\pi}{L}, \pi \qquad N_F \text{ odd}. \tag{42}$$

The lattice momentum $q$ labels the momentum creation and annihilation operators that also obey the fermion anti-commutation rules. The unitary operator $U$ has a direct product structure in terms of these fermion variables:

$$U = \mathrm{e}^{-i\frac{BL}{2}}\prod_{q>0} V_q \qquad N_F \text{ even}, \tag{43}$$

$$= \mathrm{e}^{-i\frac{BL}{2}} V_0 V_\pi \prod_{q>0} V_q \qquad N_F \text{ odd} \tag{44}$$

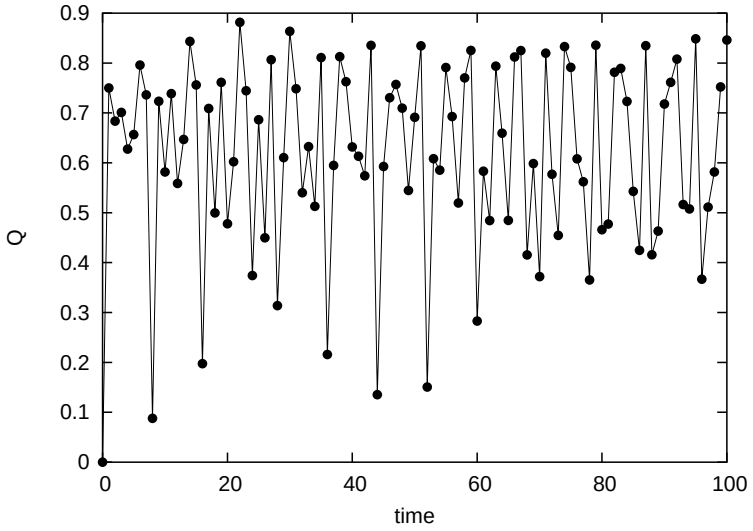


FIG. 3: The measure $Q$ for the kicked transverse Ising interaction, when the initial state is the vacuum state and $L = 10$, and the parameters are $J_x = \pi/2$, $B = \pi/3$. Shown are the results of the numerical calculations (points) and using the formula (solid line). Periodic boundary conditions are assumed.

where

$$V_q = \exp\left(-i\frac{J_x}{2}\left[\cos(q)(c_q^\dagger c_q + c_{-q}^\dagger c_{-q}) + \sin(q)(c_q c_{-q} + c_{-q}^\dagger c_q^\dagger)\right]\right) \quad \exp\left(-iB(c_q^\dagger c_q + c_{-q}^\dagger c_{-q})\right), \tag{45}$$

and

$$V_0 = \exp\left(-i(B + \frac{J_x}{2})c_0^\dagger c_0\right), \quad V_\pi = \exp\left(-i(B - \frac{J_x}{2})c_\pi^\dagger c_\pi\right). \tag{46}$$

The eigenstates of $U$ are direct products of eigenstates of $V_q$. The operators $V_0$ and $V_\pi$ are diagonal in the number basis states. For $V_q$, the four basis states are $|0\rangle, |\pm q\rangle = c_{\pm q}^\dagger|0\rangle, |-qq\rangle = c_{-q}^\dagger c_q^\dagger|0\rangle$. The eigenstates of $V_q$, for $q \neq 0, \pi$ are given by

$$V_q|\pm q\rangle = \mathrm{e}^{-i(\frac{J_x}{2}+B)}|\pm q\rangle, \quad V_q|\pm\rangle = \mathrm{e}^{-i(\frac{J_x}{2}+B)}\mathrm{e}^{\pm i\theta_q}|\pm\rangle. \tag{47}$$

Here the eigenstates $|\pm\rangle$ are given by $|\pm\rangle \equiv a_\pm(q)|0\rangle + b_\pm(q)|-qq\rangle$. Using $\cos(\theta_q) = \cos(B)\cos(J_x/2) - \cos(q)\sin(B)\sin(J_x/2)$, we have

$$a_\pm(q)^{-1} = \sqrt{1 + \left(\frac{\cos(J_x/2) - \cos(\theta_q \pm B)}{\sin q \sin B \sin(J_x/2)}\right)^2}, \tag{48}$$

$$b_\pm(q) = a_\pm(q)\frac{\pm\sin(\theta_q) + \cos(J_x/2)\sin B - \cos q \cos B \sin(J_x/2)}{\sin(q)\sin(J_x/2)}e^{-i2B}. \tag{49}$$

This then completely solves the kicked transverse Ising model. Let us consider an initial state with $m$ (even) fermions $|\psi(t=0)\rangle = |l_1, l_2 ... l_m\rangle$ where $l_i$ denote the sites occupied by fermions (corresponding to $S_{l_i}^z = 1/2$ in terms of the original spin variables). The off-diagonal matrix element of $\rho_l$ through time evolution with $U$ is

$$\langle S_l^+(t)\rangle \equiv= \langle\psi(t)|e^{i\pi\sum c_n^\dagger c_n} c_l^\dagger|\psi(t)\rangle = 0, \tag{50}$$

as the time evolution mixes only states with even number of fermions. The diagonal matrix elements of $\rho_l$ depend on $\langle S_l^z\rangle \equiv \langle\psi(t)|c_l^\dagger c_l|\psi(t)\rangle - 1/2$. This can be calculated from the time-evolved operator,

$$c_q(t) = V_q^{\dagger t}\, c_q\, V_q^t = \zeta_q\, c_q \;-\; \mathrm{sgn}(q)\,\eta_q\, c_{-q}^\dagger, \tag{51}$$

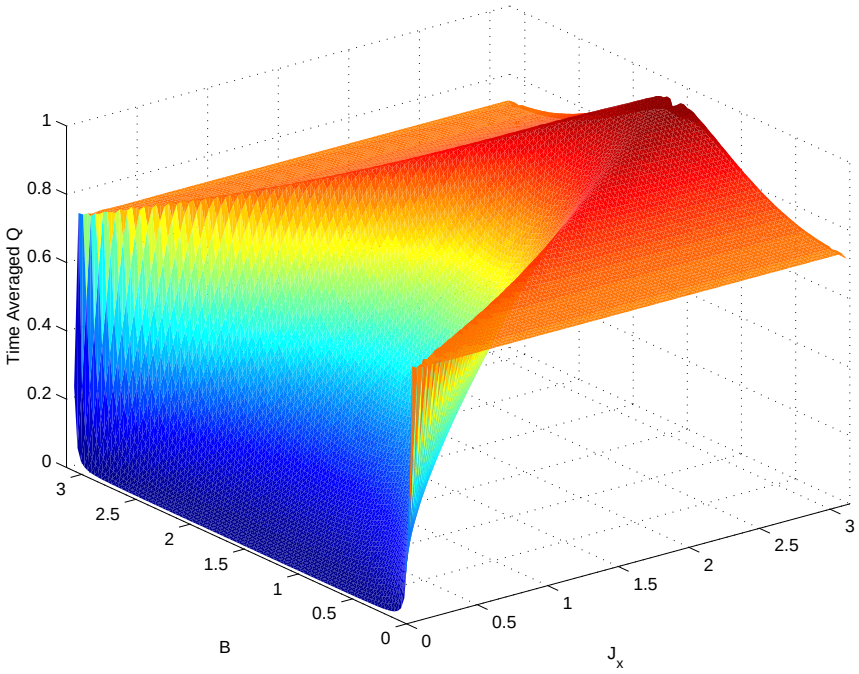


FIG. 4: (Color Online) The time averaged $Q$ as a function of system parameters for the kicked transverse Ising model. $L = 20$ in this case, and the averaging is done over a thousand kicks, by which time the average is stationary.

where the expansion coefficients are given as

$$\zeta_q = |a_+(q)|^2 e^{-it\theta_q} + |a_-(q)|^2 e^{it\theta_q}, \tag{52}$$

$$\eta_q = a_+(q)^* b_+(q) e^{-it\theta_q} + a_-(q)^* b_-(q) e^{it\theta_q}. \tag{53}$$

The diagonal matrix element can be expressed in terms of the Fourier transforms of the above functions, after some manipulations, we have

$$\langle S_l^z(t)\rangle = -\frac{1}{2} + \frac{1}{L}\sum_q |\eta_q|^2 + \sum_{i=1}^{m} |\zeta(l-l_i)|^2 - |\eta(l-l_i)|^2. \tag{54}$$

In the above we used two more auxiliary functions defined by

$$\eta(l) = \frac{2}{L}\sum_{q>0} \eta_q \cos(ql), \tag{55}$$

$$\zeta(l) = \frac{2}{L}\sum_{q>0} \zeta_q \cos(ql). \tag{56}$$

In particular for the initial unentangled state $|\psi_L(0)\rangle = |0\rangle^{\otimes L}$, as a special case we can calculate $\langle S_l^z(t)\rangle$ at any site using the above.

$$\langle S_l^z(t)\rangle = \langle\psi_L(0)|S^z(t)|\psi_L(0)\rangle = \frac{1}{L}\sum_q |\eta_q|^2 - 1/2. \tag{57}$$

Here the $q$ summation extends to both positive and negative allowed values. Hence using translational symmetry the entanglement measure $Q$ is given in this case by

$$Q(\psi_L(t)) = 4\,x\,(1-x),\ x = \frac{1}{L}\sum_q |\eta_q|^2 = \frac{4}{L}\sum_q |a_+(q)a_-(q)\sin(\theta_q\,t)|^2 \tag{58}$$

As illustrated in the example (Fig. (3)) the oscillations of $Q$ are now much more complicated. The advantage of having an easily computable formula such as Eq. (58) is that we can study the entanglement measures as a function